NUSTrive by Nowsherwan

For Lectures/Tests/FLPs - Contact 03369374191

Complex #5 NUSTrive Guide - Sol/Key

1) → D → $\frac{1+i}{1-i} \Rightarrow \frac{(1+i)(1+i)}{(1-i)(1+i)} \Rightarrow \frac{(1+i)^2}{2} \Rightarrow \frac{2i}{2} \Rightarrow i$

2) $\left((1+i)^4\right)^2 \Rightarrow (1+i^2+2i)^2 \Rightarrow (2i)^2 \Rightarrow 4(-1) = -4$

3) $i+i^2+i^3+i^4 = 0$, $i^5+i^6+i^7+i^8 = 0$ --- $i^{80} = 0$ → So 1

4) $i^{-1}+i^{-2}+i^{-3}+i^{-4} \Rightarrow 0$ --- till i^{-2020} → So last term i^{-2021} → Simplify th... ourself

5) Sum of Consecutive even/odd powers of iota = 0

So $\frac{1+0}{1+0} = 1$

6) $1+i^2+2i = x-iy$

$2i = x-iy$ Compare

$y = -2, x = 0$

7) A

8) A

9) $(i)^{3/2} \Rightarrow (i^3)^{\frac{1}{2}} \Rightarrow (-i)^{\frac{1}{2}} \Rightarrow \sqrt{-i} \Rightarrow$

or $i = \cos 90 + i \sin 90$

$i^{3/2} = (\cos 90 + i \sin 90)^{3/2} \Rightarrow \cos 90(\frac{3}{2}) + i \sin 90(\frac{3}{2})$

$\Rightarrow \cos 135 + i \sin 135$

simplify

10) 0

11) $z = x+iy$, $\bar{z} \Rightarrow x-iy$ $\left|\frac{x+iy}{x-iy}\right| = \frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}} \Rightarrow 1$

12) $z = 2$ then $\bar{z} = 2$, same, Im part is 0

13) $z = 3+4i \rightarrow z^{-1} \Rightarrow \frac{a}{a^2+b^2}, \frac{-bi}{a^2+b^2}$

$\Rightarrow \frac{3}{3^2+4^2} \quad \frac{-4i}{3^2+4^2}$

14) $z^{-1} = \frac{1}{z} \Rightarrow \frac{1}{3+4i} \Rightarrow 3+4i \Rightarrow$ Re part $\Rightarrow$

15) $z = x+iy$

so $|x+iy+1| = |x+iy-1|$

$\Rightarrow |(x+1)+iy| = |x-1+iy|$

$\Rightarrow (x+1)^2 + y^2 = (x-1)^2 + y^2$

$\Rightarrow (x+1)^2 = (x-1)^2 \Rightarrow x^2+1+2x = x^2+1-2x$

$4x = 0 \Rightarrow x = 0$

$z = x+iy$

$z = 0+iy$

$z = iy$

purely Img

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16) C

17) B

18) $\tan^{-1}\left|\frac{1}{1}\right| \Rightarrow 45^\circ$

19) A

20) C

21) A

22) $\frac{z_1}{z_2} = ki$ Since $\frac{z_1}{z_2}$ is purely Img

$$\left|\frac{z_1+z_2}{z_1-z_2}\right| \Rightarrow \left|\frac{\frac{z_1}{z_2}+\frac{z_2}{z_2}}{\frac{z_1}{z_2}-\frac{z_2}{z_2}}\right| \Rightarrow \left|\frac{ki+1}{ki-1}\right|$$

$$\Rightarrow \frac{\sqrt{k^2+1}}{\sqrt{k^2+1}} \Rightarrow 1$$

23) $(1-i)^4$

$\Rightarrow ((1-i)^2)^2$

$\Rightarrow (-2i)^2 \Rightarrow 4(-1) = -4$

24) $-i \Rightarrow i^3 \Rightarrow (i^3)^{\frac{1}{3}} \Rightarrow i$

25) $\tan^{-1}\left|\frac{\sqrt{3}}{1}\right| \Rightarrow \tan^{-1}\left|\frac{\sqrt{3}}{1}\right| \Rightarrow 60^\circ$

26 Its in 2nd Q so Arg $= 180 - \tan^{-1}\left|\frac{y}{x}\right|$

$\Rightarrow 180 - 60 \Rightarrow 120$

26) 25) $|i| \Rightarrow 1$

$\text{Arg}(1) = 0^\circ$

27) $\text{Arg}(-1-i) \Rightarrow \tan^{-1}\left|\frac{-1}{-1}\right| \Rightarrow \tan^{-1}\left|\frac{1}{1}\right| \Rightarrow 45^\circ$

3rd Q so $\text{Arg} = \tan^{-1}\left|\frac{y}{x}\right| - 180$

$\Rightarrow 45 - 180 \Rightarrow -135$

28) D

29) $\frac{\pi}{2}, \frac{-\pi}{2}$ both

30) $z = x+iy \Rightarrow |x+iy| = 1$

$x^2+y^2 = 1 \rightarrow B$

31) $i^1 = i, i^2 = -1, i^3 = -1, i^4 = 1, i^5 = i$

So 4 values $i, -1, -i, -1$

32) 4 $\rightarrow B$

33) $|(1+i)^n| \Rightarrow |(1+i)|^n$

$|(1+i)^{10}| \Rightarrow |(1+i)|^{10} \Rightarrow \sqrt{1^2+1^2} \Rightarrow (\sqrt{2})^{10} \Rightarrow 2^5$

34) $e^{\ln f(x)} = f(x) \rightarrow e^{\ln i^2} \Rightarrow i^2 \Rightarrow -1$

34) repeated

$\Rightarrow |a| \cdot |b| \Rightarrow |a \cdot b|$

$\Rightarrow \left|\frac{1+i}{1-i} \times \frac{1-i}{1+i}\right| \Rightarrow |1| \Rightarrow 1$

$\sqrt{9}\cdot\sqrt{b} = \sqrt{9b}$ and $\frac{\sqrt{9}}{\sqrt{b}} = \sqrt{\frac{9}{b}}$

35) $\frac{\sqrt{5}\cdot\sqrt{2}\cdot\sqrt{2}\cdot 4}{4\sqrt{2}\cdot\sqrt{5}\cdot\sqrt{5}} \Rightarrow \frac{\sqrt{10}}{\sqrt{25}} \Rightarrow \sqrt{\frac{10}{25}} \Rightarrow \sqrt{\frac{2}{5}} \Rightarrow \frac{\sqrt{2}}{\sqrt{25}}$

36 purely img, real part = 0

$k+2=0$, $k=-2$

37) purely real, Imgpart = 0, $m-n=0$

38) Simplify

39) $z = x+iy$, $x+iy+x-iy=0$

$2x=0 \Rightarrow x=0$

$z = 0+iy \Rightarrow$ purely Img

39) B

40) A

41) $Arg\left(\frac{a}{b}\right) = Arg\,a - Arg\,b$

or $\frac{1+i}{1-i} \Rightarrow \frac{1+i}{1-i}\frac{(1+i)}{(1+i)} \Rightarrow i \Rightarrow Arg(i) = 90^\circ$

42) A

43) $e^{ix} = \cos x + i\sin x \rightarrow e^{i\pi} = \cos\pi + i\sin\pi \Rightarrow -1 + i(0) \Rightarrow -1$

44) $|-1| \Rightarrow 1$

45) D (simplify options)

46) $x = r\cos\theta$, $y = r\sin\theta$ $\quad (1, \sqrt{3}0)$ (r, θ)

47) A

48) Next Page

48) $\cos\theta + i\sin\theta$ is standard form

$i\cos\theta + \sin\theta \Rightarrow \frac{i}{i}(i\cos\theta + \sin\theta)$

$\Rightarrow \frac{1}{i}(-\cos\theta + i\sin\theta) \Rightarrow -\frac{1}{i}(\cos\theta - i\sin\theta)$

$-\frac{1}{i}(e^{i-\theta})$ D) None

49) $\frac{0+i}{0-i} \Rightarrow -1$

or 

$\cos 90 - i\sin 90$

$\Rightarrow \cos(-90) + i\sin(-90)$

So $\frac{\cos 90 + i\sin 90}{\cos(-90) + i\sin(-90)} \Rightarrow \frac{e^{\frac{i\pi}{2}}}{e^{i(-\frac{\pi}{2})}} \Rightarrow e^{\frac{i\pi}{2}} \cdot e^{\frac{i\pi}{2}} \Rightarrow e^{\frac{i\pi}{2}+\frac{i\pi}{2}}$

$\Rightarrow e^{i\pi} \Rightarrow -1$

50) D

51) $1+i \Rightarrow |1+i| \Rightarrow \sqrt{2} \Rightarrow r$

$\theta = \tan^{-1}\left|\frac{1}{1}\right| \Rightarrow \frac{\pi}{4} \Rightarrow r(\cos\theta + i\sin\theta)$

$\Rightarrow \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$ → Ans D (trig used in C)

52) $\theta = 90, n = 10$

$(\cos 90 + i\sin 90)^{10} \Rightarrow (0+1)^{10} \Rightarrow 1$ ✓

53) B

54) A

55) $\cos\theta + i\sin\theta = e^{i\theta}$

$\cos\theta - i\sin\theta \Rightarrow \cos(-\theta) + i\sin(-\theta) \Rightarrow e^{-i\theta}$ ✓

$\bar{z}$ has Argument $-\theta$

56) B

57) $Arg(z^n) = n\,Arg(z)$

$\Rightarrow$ $arg\,z^4 \Rightarrow 4\,arg\,z \Rightarrow 4(\pi)$ $\rightarrow$ Not principle

so $4\pi \pm 2m\pi$

put $m=-2$

$4\pi + 2(-2)\pi \Rightarrow 0\pi$ principle ✓

58) $(0+1)^{12} \Rightarrow i \Rightarrow 0+1$

$x=0, y=1 \quad \frac{x}{y} \Rightarrow 0$

59) Triangle ~~Inequality~~ theorem

$|z_1+z_2| \leq |z_1|+|z_2|$

60) $||z_1|-|z_2|| \leq |z_1+z_2| \leq |z_1|+|z_2|$ $\quad z_2 = 3+4i$, $|z_2| \Rightarrow 5$

$|3-5| \leq 11 \leq 3+5$

$2 \leq 11 \leq 8 \rightarrow \frac{8}{2}$

61) Take ~~Square~~ one of options, whichever gives $3+4i$ is Ans

62) $z^2+1=2z$

$z^2-2z+1=0 \rightarrow$ Quadratic

63) $z^2+2=2z \rightarrow z^2-2z+2=0$

Quadratic

64) $\sum_{n=0}^{12} i^n + \sum_{n=1}^{12} -i^n$

$\Rightarrow i^0 + i^1 + i^2 - i^{12} + (-1) \sum_{n=1}^{12} i^n$

$\Rightarrow i^0 + 0 - \sum_{n=1}^{12} i^n$

$\Rightarrow 1 + 0 - [i + i^2 + i^3 - - - i^{12}]$

$\Rightarrow 1 + 0 - [0] \Rightarrow 1$

65) Multiply opt

or $x^2 + 3 = x^2 - (-3)$

$x^2 - (\sqrt{3}i)^2$

$(x + \sqrt{3}i)(x - \sqrt{3}i)$

66) No Comparison of greater Smaller in Complex Num D

67) $(1-3i)^{-1} \Rightarrow \frac{1}{1-3i}\left(\frac{1+3i}{1+3i}\right) \Rightarrow \frac{1+3i}{1^2-(-9)} \Rightarrow \frac{1+3i}{10} \Rightarrow \frac{1}{10} + \frac{3i}{10}$

68) $\frac{(1+i)^8}{(\sqrt{2})^8} \Rightarrow \frac{(1+i)^2)^4}{2^4} \Rightarrow \frac{(1+i^2+2i)^4}{2^4} \Rightarrow \frac{2^4 \cdot i^4}{2^4} \Rightarrow 1$

68) A

70) $\left| \frac{1+i}{1-i} - \frac{1-i}{1+i} \right| \Rightarrow \frac{a \quad b}{b \quad a}$ LCM ShortCut

$\frac{a(a) + b(b)}{ab}$

So $\left|\frac{(1+i)^2-(1-i)^2}{(1+i)(1-i)}\right|$ → simplify, take mod

71) $\frac{1+i}{1-i} = i \rightarrow i^m = 1 \rightarrow m = 4$

72) $i^{4k} = 1$, $\frac{i^{4k} - i^{4k} \cdot i^{-1}}{i^{4k}} \Rightarrow \frac{i^{4k}}{i^{4k}} - \frac{i^{4k} \cdot i^{-1}}{i^{4k}}$

$1 - i^{-1} \Rightarrow 1 - \frac{1}{i} \Rightarrow 1 - (-i) \Rightarrow 1+i$

73) z mirror image is its conjugate

So $-1+i$

74) If i and ki are reciprocal then their product is 1

$i(ki) = 1 \rightarrow ki^2 = 1 \rightarrow k = -1$

75) $\left(\frac{2+2i+2-2i}{2}\right)^4 \Rightarrow \left(\frac{4}{2}\right)^4 \Rightarrow 2^4 \Rightarrow 16$

76) A

77) wrong Qs

78) purely Img e.g. $2i$, then $\bar{z}$ is $-2i$

$\bar{z} = -z$ since $2i = -(-2i)$

79) $\overline{\sqrt{-3}} \Rightarrow \overline{\sqrt{3}i}$

$\overline{\sqrt{-4}} \Rightarrow \overline{\sqrt{4}i}$ $\rightarrow \overline{\sqrt{3}i} \cdot \overline{\sqrt{4}i} \Rightarrow i^2 \cdot \overline{\sqrt{3} \cdot \sqrt{4}}$

$\Rightarrow -1 \overline{\sqrt{3} \cdot 4} \Rightarrow -1 \overline{\sqrt{12}}$

80) $\sqrt{-3.4} \Rightarrow \sqrt{-12} \Rightarrow \sqrt{12}\cdot\sqrt{-1} \Rightarrow \sqrt{12}\,i$

81) $(1,0) \Rightarrow 1+0i$

82) wrong → correct Qs is

$$\text{Arg}(z)+\text{Arg}(-z)=?$$

Put $z=i$

$$\text{Arg}(i)+\text{Arg}(-i)$$

$$\Rightarrow 90+(-90)=0$$

83) Check properties in book or Asume $z=i$ and check if Given Statement Correct or not

84) fin $|z|.z_2$ then its real part

85) replace i with $-i$

$$\Rightarrow \frac{-(-i)}{1+(-i)} \Rightarrow \frac{i}{1-i}$$

86) $\overline{\left(\frac{z}{i}\right)} \Rightarrow \frac{\bar{z}}{\bar{i}} \Rightarrow \frac{\bar{z}}{-i} \Rightarrow \frac{\bar{z}(-i)}{-i(-i)} \Rightarrow \frac{-i\bar{z}}{-1} \Rightarrow i\bar{z}$

87) $\bar{z} = -\frac{1}{2}+\frac{\sqrt{3}}{2}i$ then $z+\bar{z}\bar{i}^{1}=0$

simplify

88) $\bar{z}=z^{-1} \rightarrow \bar{z}=\frac{1}{z} \Rightarrow z\cdot\bar{z}=1$ → D

$z\cdot\bar{z}=|z|^2 \rightarrow$ so $|z|^2=1$

$|z|=1$

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89) A

90) $e^{\ln f(x)} = f(x) \rightarrow e^{\ln i} = i$

91) $z = 3+4 \rightarrow z = 7 \rightarrow |z| = 7$

92) $3-4 \Rightarrow -1 \rightarrow \bar{z} \Rightarrow -1$

$z = -1+0i \rightarrow \bar{z} \Rightarrow -1+0(-i) \Rightarrow -1$

93) $i = e^{i\frac{\pi}{2}}$

$\left(\frac{i}{2}\right) = \left(e^{i\frac{\pi}{2}}\right)^{\frac{1}{2}} \Rightarrow e^{i\frac{\pi}{2}} \rightarrow e^{\frac{-\pi}{2}} \Rightarrow \frac{1}{e^{\frac{\pi}{2}}} \rightarrow \frac{1}{\sqrt{e^{\pi}}}$

94) B

95) B

96) B

97) A

98) B

99) $\tan^{-1}\left|\frac{-\sqrt{3}}{-1}\right| \xrightarrow{\text{Mod}} \tan^{-1}\left|\frac{\sqrt{3}}{1}\right| \Rightarrow \tan^{-1}(\sqrt{3}) = 60^\circ$

4th ⊕ so $\text{Arg} = -\tan^{-1}\left|\frac{y}{x}\right| \Rightarrow -60^\circ$

100) $x^3 \geq 0$ always $\rightarrow$ so D is wrong